News and Notes

Mathematical Modeling of Archaeological Artefacts from Keezhadi Excavation, Tamil Nadu, South India

K. Kaliraj[1], K. Jeganathan[1], M. Manjula[1], S. Selvakumar[1] and S. M. Hussain[2]
[1] *Ramanujan Institute for Advanced Study in Mathematics, University of Madras,Chepauk, Chennai -600 005(TN), India*
[2] *Department of Geology, School of Earth and Atmospheric Sciences, University of Madras, Guindy Campus, Chennai-600 025(TN), India*
*(*Corresponding Author, E-mail:kaliraj@unom.ac.in)*

Keezhadi Excavation site is located in the south of Krishna and Tungabhadra rivers in Tamil Nadu, India. The Archaeological Survey of India (ASI) retrieved a lot of artifacts like dice, pottery items, gold coins etc which belongs to the Sangam Period (3rd century B.C. to the 3rd century A.D.) in Southern India. The major geo-archaeological techniques examined in the present study are: 1) discovering archaeological sites and documenting their internal structures, 2) surveying site formation and disturbance processes, 3) analyzing soils and sediments, 4) paleo-environmental reconstruction and the impact of humans (anthropogenic) on the landscape, 5) physical analysis of archaeological materials and 6) integrating geo-archaeology with social archaeology.

Keezhadi or Keeladi, site is located near the town of Keezhadi in Sivaganga District, Tamil Nadu, about 12 km southeast of Madurai. The Keezhadi Excavation was carried out in eight phases. ASI began the initial phase of excavation at Keezhadi's vicinity of the Vaigai River in 2015. Subsequently, in 2016, the second phase started. During this phase, over 6,000 items were discovered which included kitchen wells, government seals etc. The third phase of excavation, which was conducted by the ASI in 2017. Eighty acres of land were chosen for 16 digging sites in the third phase. The 4th phase of excavation was completed during 2017 and 2018 in which 5,820 samples were retrieved. The bricks from the Sangam Period and more than 700 other artifacts were discovered during the fifth round of excavation from 2020 to 2022, phase wise all the remaining excavations were completed

Numerous ancient buildings and artefacts have been discovered in over 48 square pits, including skeleton tools, roof tiles, etc. Many of these samples were also sent to various archaeological institutes to confirm the age using the radiocarbon dating methods. However, this cannot be possible for all the samples because every sample differs in the process of their deposition. Mathematics and computer science act as a main tool to illustrate the hereditary properties of various samples that were found in Keezhadi Excavations, and also to view more about the progress of the deposition of archaeological samples (Brughmans and Peeples, 2023; King, 2015; Looper, 2024; Peeples, 2019).

We discuss the basic history of architecture and mathematics and focuses on the applications of the mathematical tools on different samples of Keezhadi area, Tamil Nadu.

History of Architecture and Mathematics

Archaeology, like the majority of scientific fields, underwent extensive research in the use of computers and mathematics since 1950. From 1945 to 1950, the development of quantitative movement in anthropology, sociology, and psychology between the two world wars had a subtle but significant influence on archaeology (Franois Djindjian, 2014). However, in 1945, the scientists who involved in the process of war moved their attention back to basic studies, *viz.*, and people began utilizing the methods and tools developed during the fight, placing a focus on quantitative approaches and the use of mathematics (Franois Djindjian, 2014). The field of archaeology had an explosion in quantification between 1950 and 1965, employing basic statistics and graphics which constitute the basis of the methodologies used to process archaeological material (Franois Djindjian, 2014). The computer freed the Archaeologists from hand estimation in 1965 (Franois Djindjian, 2014). There are nearly six case studies and approaches to analyze the methods of archaeology. However, in the 5th case study, numerous applications of mathematical modeling approaches, such as algebraic and exponential equations, gravity models, linear programming, *etc.*, were made in the field of archaeology. Therefore, for this study, some recent secondary samples were collected in the Keezhadi, and by using mathematical tools, mean life, and half-life of the samples were determined. From this study, a detailed result at architecture and mathematics can be obtained (John *et al.*, 2015; Juan *et al.*, 2015; Laura, 2008; Wilkinson, 1971).

Mathematical Findings Using Secondary Samples in Recent Times

A study has been made on different samples of Keezhadi (Rectangular Ivory Dice, Coconut plantation with a pottery yard, silver punch-marked coin, Iron tools, gold coins) by applying the mathematical tools, to obtain the desired results. By Mathematical tool, we determine the age of the samples discovered at Keezhadi using the carbon-14 dating method, (Kapur, 2013; Kapur, 1985; Olink, 1978). These articles (Read, 1989; Renfrew and Cooke, 2014) are referred for studying more about statistical methods and for reasoning out in archaeological research.

A Rectangular Ivory Dice

At the Keezhadi ancient site in Madurai, ASI discovered a rectangular dice made of ivory which measures 4.5 cm x 0.9 cm x 0.9 cm (Fig. 1). To denote the numbers on the dice, residents of Keezhadi utilized circles. (The times of India, 19th February, 2022, Madurai).

The volume, surface area and total surface area of the rectangular ivory dice with the help of the given data using the mathematical process were obtained.

Volume of a ivory dice $= L \times B \times H$ cubic units
$= 4.5 \times 0.9 \times 0.9\ \text{cm}^3 = 3.645\ \text{cm}^3$

Total Surface area of an ivory dice $= 4 \times LB + 2 \times BH\ \text{cm}^2$
$= 16.2 + 1.62\ \text{cm}^2 = 17.82\ \text{cm}^2$

Carbon Dating Confirms Sangam Period for Keezhadi Site

At Keezhadi site, a pottery yard in the centre of a coconut plantation was excavated (Fig. 2). Carbon-14 age of

(Received: 03 July 2024; Revised from accepted: 25 March 2025) https://doi.org/10.56153/g19088-024-0210-73

Fig.1. Rectangular ivory dice

Fig.2. Coconut Plantations

the two samples from this site is around 2,200 years old. (The Hindu, 2017).

Radioactive Decay: Half-Life

The Half-life of a radioactive element is the time required for half of the radioactive nuclei decay present in a sample, and is independent of the number of initially nuclei present.

The decay of the radioactive element occurs exponentially, according to the differential equation $dy/dt = ky$ where k is some negative constant.

Mean-Life

The mean lifetime of a radioactive element is reciprocal of its decay constant.

That is, $\tau = 1/k$ (or) Mean-life = 1.443×Half-life. Half-life is 50% decay of original atoms present initially while average life is 63% decay of initial atoms. Let y_0 be the initial quantity of radioactive nuclei. The quantity y of nuclei existing at time t is thus determined by $y = y_0e^{kt}$. Let's know the time t at which only 1/2 (y_0) nuclei are still present since we are trying to determine the half-life by $t = In2/-k$. Since $In2/-k$ is +ve. Therefore on k determines the half-life. It's important to remember the following formula for prospective usages: half-life = $In2/-k$.

Let's demonstrate the age of the potteries that were discovered to have 90% of their original radioactive Carbon 14 content (Kapur, 2013).

Verification

From the equation $y = y_0e^{kt}$ two things can be determined: (*i.e.*) the value of k and t.

For which $y_0e^{kt} = (90/100)\,y_0$, *i.e.*, find *t such that* $e^{kt} = 9/10$. Re-arranging the half-life equation and utilizing the fact that the half-life is 14,470 years,

$$-k = (In2/\text{half - life})$$

$$-k = In2/14470 \approx 0.0000479$$

$$k = -0.0000479.$$

To evaluate t which makes $e^{kt} = 9/10$

$$e^{-0.0000479t} = 0.9$$

$$-0.0000479t = In(0.9)$$

$$t = \frac{In(0.9)}{-0.0000479} \approx 2199 \text{ yrs.}$$

The Half-life and Mean life of the radioactive decay and also verified the original amount of radioactive ^{14}C in the age of the potteries were obtained by utilizing the Mathematical process. From this study, it is observed that the sample is approximately 3rd century BC.

Silver Punch-Marked Coin

On 04th August, 2021, a punch-marked silver coin was discovered at the Keezhadi Excavation site in Sivaganga District of Tamil Nadu (Fig. 3). This discovery suggests that people from the area traded with north India using currencies that were common in the sixth century BCE. According to the then archaeology minister Mr. Thangam Thennarasu, the coin's geometric markings and images of the sun, moon, bull, taurine, and animal figurine demonstrate that the Keezhadi people were a trade society. The punch-marked coin was discovered at a depth of 146 cm and is thought to represent proof of commerce between the north and south, notably with the Gangetic Valley. From the period of the Mahajanpadas in the 6th century BCE, silver punch-marked coins were in use (The Hindu, 04th August, 2021).

From the above given information, the findings may be presented as follows:

Let's find the half-life and mean-life of a 6th century BCE

Fig.3. Silver punch-marked coin

artefact that was discovered to contain 90% of its original radioactive carbon 14 content (T.N. Kapur, 2013).

From the equation $y = y_0 e^{kt}$ two things can be determined: (*i.e.*) the value of k and t. For which $y_0 e^{kt} = (90/100)\, y_0$, *i.e.*, find t such that $e^{kt} = 9/10$.

1. Evaluate the value of k

If the half-life is known to be 17114 years, rearranging the half-life equation and utilising this information, $k = -0.0000405$

2. Evaluate the value of t which makes $e^{kt} = 9/10$

$$e^{-0.0000405t} = 0.9$$
$$0.0000405t = In(0.9)$$
$$t = \frac{\text{In}(0.9)}{-0.0000405} \text{ » } 2199 \text{ yrs » } 6 \text{ century BCE.}$$

It is concluded that,

(i) the Half-life of a silver punch-marked coin is approximately years.

(ii) the Mean-life of a silver punch-marked coin is approximately years.

Krishnagiri Site Traces Iron Tools to 2172BCE

The iron implements discovered on 10th May, 2022, are traced to the Krishnagiri site (Fig. 4). One of the nation's oldest iron ages, Tamil Nadu has a 4,200-year-old iron age. On 9th May, 2022, the Tamil State Archaeology Department published a report, in which it is mentioned that the Tamil civilization had mastered the technique of creating iron tools, which had a positive economic impact since they allowed for the cultivation of bigger regions than the stone tools (The Times of India, 2022).

The half-life and mean-life of a piece of art that was discovered contain 90% of its original radioactive carbon 14 content if it dated from 2,172 BCE (Kapur, 2013).

Half-life

Utilizing the fact that the half-life is about 27,615 years and rearranging the half-life calculation.

Mean-life

From the half-life of iron tools, a mean-life as $\approx$ 39,848 years, was obtained.

$$-k = (In2/\text{half - life})$$
$$-k = In2/27165 \approx 0.0000251$$
$$k = -0.0000251.$$

To evaluate t which makes $e^{kt} = 9/10$

$$e^{-0.0000251t} = 0.9$$
$$-0.0000251t = \text{In}(0.9)$$
$$t = \frac{\text{In}(0.9)}{-0.0000251} \approx 4{,}197 \text{ yrs} \approx 4{,}200 \text{ yrs}$$

The half-life and mean-life of an iron tool were obtained with the help of the given data via the Mathematical process. From this study, it is inferred that the sample is approx. 4,200 years old.

Gold Coins and Jewellery

Archaeologists have found gold at the Keezhadi site, near Madurai. Verification of several gold coins and decorations found at this site provided insight into the historic Tamil Civilization and Tamil Culture (Fig. 5). Keezhadi has seen the discovery of a few gold coins and jewellery items. In Keezhadi, the ASI conducted 3 rounds of excavation and collected over 8,300 artefacts. These artefacts are 2,200 years old as per the carbon dating. The Tamil Nadu Archaeology Department started the 4th phase of excavation recovered valuable items. Subsequently, an additional 1.5 acres of land was acquired near the current location and 16 trenches were dug to continue the excavation in Keezhadi and produced fascinating findings. According to the old history, gold jewellery was discovered in two trenches at the first location for the first time (Deccan Chronicle, 2018).

Let's verify the gold coins age by demonstrating that they have 90% of their original radioactive Carbon 14 content (Kapur, 2013).

Verification

Two things can be illustrated by using the equation $= y_0 e^{kt}$

1. Evaluate the value of k

For which $y_0 e^{kt} = (90/100)\, y_0$ *i.e.*, find t such that $e^{kt} = 9/10$. If the half-life is known to be 18483 years, rearranging the half-life equation and utilising this information,

$$-k = (In2/\text{half - life})$$
$$-k = In2/18483 \approx 0.0000375$$
$$k = -0.0000375.$$

Fig.4. Iron Tools

Fig.5. Gold Coins

2. Evaluate the value of t which makes $e^{kt} = 9/10$

$$e^{-0.0000375t} = 0.9$$

$$-0.0000375t = In(0.9)$$

$$t = \frac{\text{In}(0.9)}{-0.0000375} \approx 2{,}223 \text{ yrs} \approx \text{3rd century BCE}$$

The age of a gold coin was obtained and verified the original amount of radioactive carbon 14 by the Mathematical process.

Conclusions

An attempt has been made to determine the age of the sample discovered at Keezhadi using the carbon-14 dating method. The results of several samples collected from the ancient site of Keezhadi were analyzed mathematically to determine the Volume, Surface area, Total surface area of the rectangular ivory dice. Also, obtained the Half-life and Mean life of the radioactive decay and also verified the original amount of radioactive ^{14}C in the age of the potteries through Mathematical process. Then, the half-life and mean-life of the silver punch-marked coin and Iron tools were computed with the help of the given data via the Mathematical methodology. As a sequel, the age of a gold coin computed is verified with the original amount of radioactive ^{14}C. This Keezhadi archeological site with the available rectangular ivory dice, Coconut Plantations, Silver punch-marked coin, Iron tools and gold coins also from the age obtained belongs to the Late Holocene period. Therefore, it is justified that the various mathematical techniques can be utilized as a reliable proxy to determine the age of the samples.

Acknowledgments

The Authors would like to thank the Rashtriya Uchchatar Shiksha Abhiyan (RUSA 2.0) scheme under Ministry of Human Resource Development, Government of India awarded to the University of Madras for funding this research.

References

Brughmans, T., and Peeples, M. (2023). Network Science in Archaeology: Cambridge University Press, Cambridge.

Deccan Chronicle. (2018). 21st June, 1:39 am, Tamil Nadu.

Franois Djindjian. (2014). A Short History of the Beginnings of Mathematics inArchaeology. Proceedings of the 42nd Annual Conference on Computer Applications and Quantitative Methods in Archaeology, pp.65-85.

John Wilcock, Doran, J.E. and Hodson, F.R. (2015). Mathematics and computers in Archaeology, Edinburgh University Press, v. 51(202), pp.1-6.

Juan A. Barcel, Katia F. Achino, Igor Bogdanovic, Giacomo Capuzzo, Florencia Del Castillo, Vera Moitinho de Almeida and Joan Negre (2015). Measuring, Counting and Explaining: An Introduction to Mathematics in Archaeology-I, DOI: 10.1201/b18530-3, pp. 3-64.

King, E. (*Ed.*) (2015). *T*he Ancient Maya Marketplace: The Archaeology of Transient Space: University of Arizona Press, Tucson.

Kapur, T.N. (2013). Mathematical Modeling, Wiley Eastern Limited, New Age International Pvt. Ltd.

Kapur, T.N. (1985). Mathematical Models in Biology and Medicine, Affiliated East-West Press, New Delhi.

Olink, R. (1978). Mathematical Models in Social and Life Sciences.

Laura Tedeschini-Lalli, Elisa Conversano and Alessandra Carlini. (2008), Mathematics and Archaeology, Jour. Applied Mathemat., v. 1(2), pp. 61-68.

Looper, M. (2024). From History to Cultural Diversity: The Changing Roles of the Maya Script as Archaeological Data. *J*our. Archaeol. Res., doi.org/10.1007/s10814-024-09202-9.

Peeples, M. (2019). Finding a place for networks in archaeology: Jour. Archaeol. Res., v. 27, pp. 451–499.

Read, D.W. (1989). Statistical methods and reasoning in archaeological research: A review of praxis and promise. Jour. Quant. Archaeol.

Renfrew, C. and Cooke, K.L. (2014). Transformations: mathematical approaches to culture change. Academic Press.

The Hindu (2017). 28th July,10:31 am, IST - Chennai.

The Hindu (2021). 04th August, 02:28 am, IST - Madurai.

The Times of India (2022). 19th Feb, 04.15 pm, IST- Madurai.

The Times of India (2022). 10th May, 09:20 am, IST-Tamil Nadu.

Wilkinson, E.M. (1971). Archaeological seriation and the travelling salesman problem, Edinburgh: Edinburgh University Press, 276283.